Maximal Independent Sets in Radio Networks

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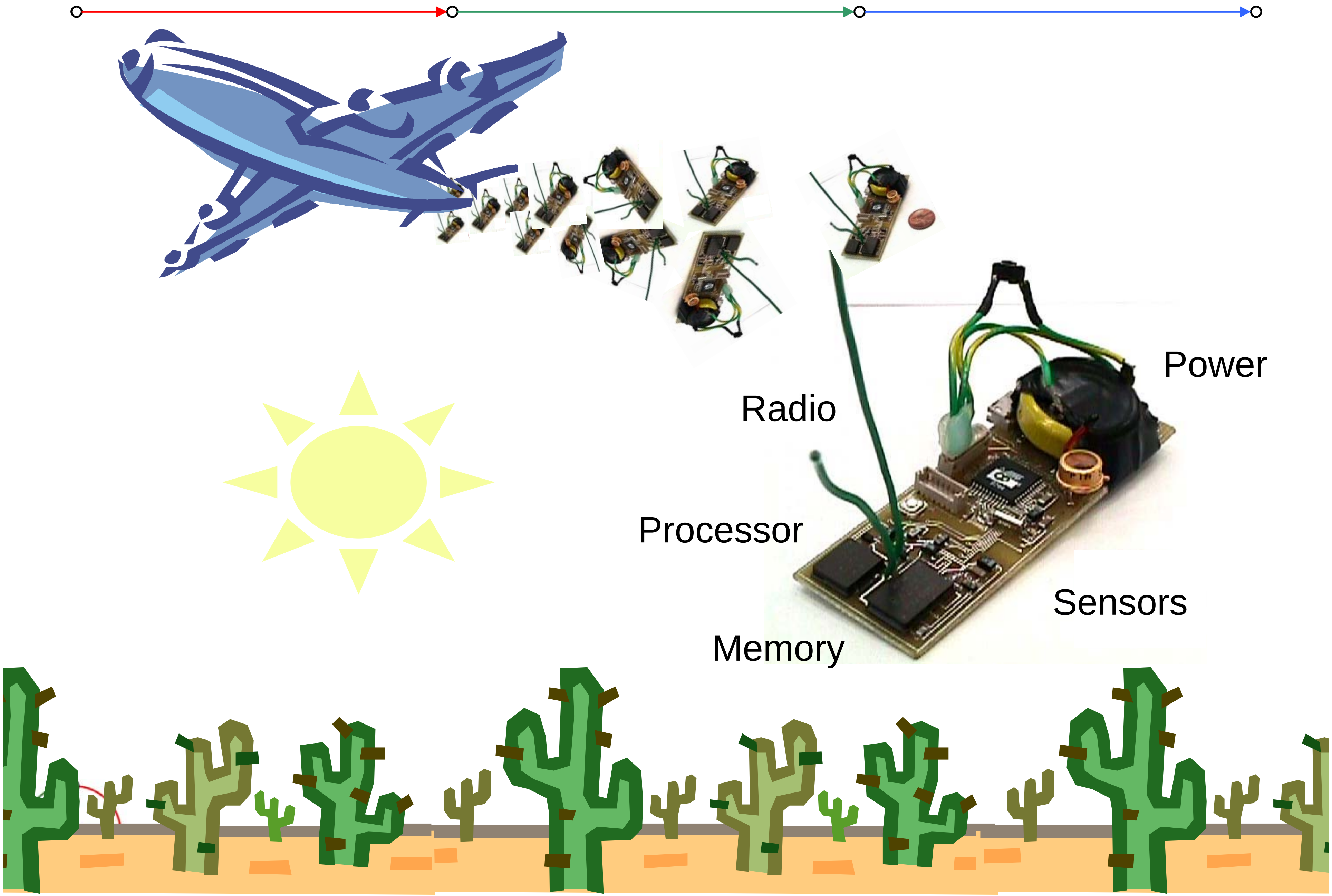
Wireless Ad Hoc and Sensor Networks

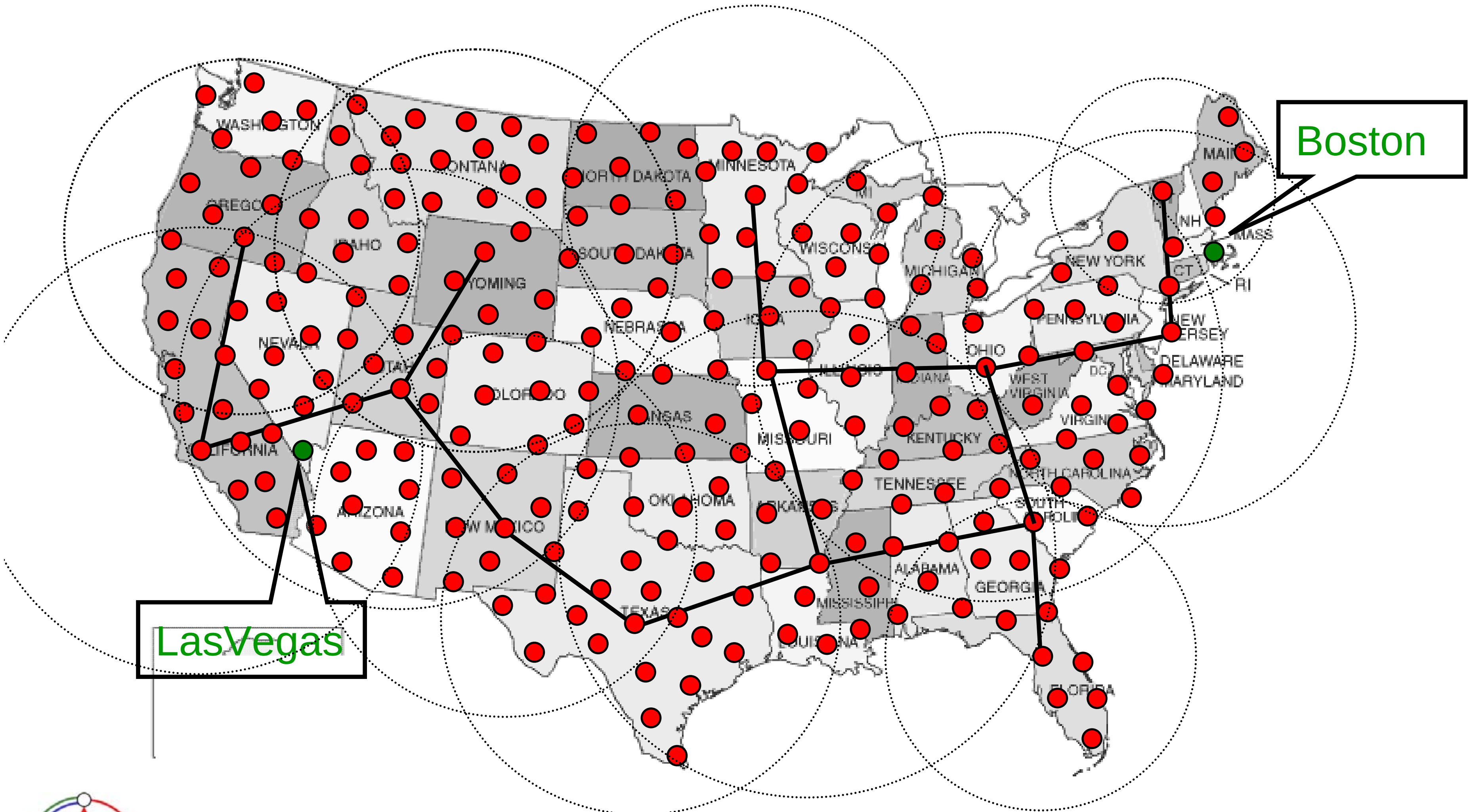


The Importance of Being Clustered... (Part A)

- Communication between distant nodes
  - Because of mobility, maintaining routing-tables is expensive
  - Hence, flooding is often a key component of routing protocols
  - Flooding is inefficient in dense networks

→ Idea: Routing on Virtual Backbones

Virtual Backbone Routing



The Importance of Being Clustered... (Part A)

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→ Idea: Routing on Virtual Backbones

1) Route to your clusterhead

2) Clusterhead routes message to clusterhead of destination

3) Clusterhead sends message to destination

compute clusters & connect clusterheads!

In this talk!

The Importance of Being Clustered...

- In wireless multi-hop networks,...
- ... clustering is important for structuring the network.

- Particularly, clustering helps in...

A) ...facilitating communication between distant nodes
  - Backbone routing

B) ...organizing communication between adjacent nodes
  - MAC layer, spatial multiplexing
  - Cluster-Leader can assign collision-free time-slots

C) ...improving energy efficiency
  - Synchronized Sleep/Awake schedules within a cluster
  - Only cluster-leader needs to remain awake

Maximal Independent Set (MIS)

- Clustering: Choose clusterhead such that:

  a) Each node has a clusterhead in its communication range.

  b) No two clusterheads interfere

- When modeling the network as a graph G=(V,E), this leads to the well-known Maximal Independent Set (MIS) problem.
- A Maximal Independent Set (MIS) is a subset $S \subseteq V$ of the nodes s.t.
  a) No two nodes $u,v \in S$ are neighbors.
  b) Every node $w \in V \setminus S$ has at least one neighbor in S.

In this talk, we study the distributed complexity of computing a MIS in a radio network model.

Overview

- Clustering in Ad Hoc and Sensor Networks
- Related Work
- Model
- Algorithm & Analysis
- Conclusions & Open Problems

MIS - Related Work (MIS)

- Distributed complexity of MIS is of fundemantal interest
  - Prototypically captures the notion of symmetry breaking
  - Numerous important applications

- Upper Bounds in general graphs
  - Randomized algorithm time O(log n) [Luby 86]
  - There are deterministic algorithms running in $o(n^c)$ for any c>0. [Awerbuch, Goldberg, Luby, Plotkin, FOCS 89], [Srinivasan, Panconesi, STOC 92]
  - Whether there is a deterministic O(polylog n) MIS algorithm is an outstanding open question in distributed complexity. [Linial 92]

- Lower Bound in general graphs
  - Every MIS algorithm requires at least time $\Omega\left(\sqrt{\frac{\log n}{\log\log n}}\right)$ [Kuhn, Moscibroda, Wattenhofer, PODC 2004]

MIS - Related Work (MIS)

- General graphs may be a too pessimistic model for describing wireless networks
- MIS also studied in restricted graph models

  (Unit Disk Graphs, Unit Ball Graphs,...)

- When nodes know the distances, MIS in time $O(\log^* n)$

  [Kuhn, Moscibroda, Wattenhofer, PODC 2005]
- Deterministic without distance-information in time $O(\log \Delta \cdot \log^* n)$.

  [Kuhn, Moscibroda, Nieberg, Wattenhofer, DISC 2005]
- Lower Bound $\Omega(\log^* n)$ [Linial 92]

All of these algorithms work in

message passing models

MIS - Related Work (Radio Networks)

- In radio networks: Much work on communication primitives (broadcast, wake-up, gossiping,…)
- Much less is known about computation of local network structures.
- In single-hop radio networks → wake-up problem!
  - Lower Bound of $\Omega(\log^2 n / \log\log n)$ [Jurdzinski, Stachowiak, ISAAC 02]
  - This lower bound also holds for the MIS case!
- In multi-hop case
  - Many linear time algorithms [Wan, Alzoubi, Frieder, INFOCOM 02], …
  - $O(\log^2 n)$ algorithm when nodes wake up simultaneously and know 2-hop neighborhood [Gandhi, Parthasarathy, FSTTCS 04]
  - $O(\log^3 n)$ algorithm using three communication channels. [Moscibroda, Wattenhofer, MASS 04]
  - $O(\Delta \log n)$ algorithm for $O(\Delta)$ coloring. [Moscibroda, Wattenhofer, SPAA 05]

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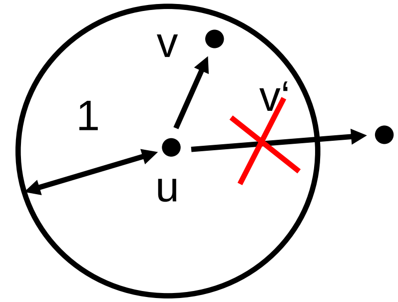
Unstructured Radio Network Model

- Multi-Hop
- No collision detection
  - Not even at the sender!
- No knowledge about (the number of) neighbors
- Asynchronous Wake-Up
  - Nodes are not woken up by messages !
- Unit Disk Graph (UDG) to model wireless multi-hop network
  - Two nodes can communicate iff Euclidean distance is at most 1
- Upper bound n for number of nodes in network is known
  - This is necessary due to Ω(n / log n) lower bound
    [Jurdzinski, Stachowiak, ISAAC 2002]



Our algorithm computes a MIS in time $O(\log^2 n)$ with high probability.

Optimal up to O(loglog n) factor

Time-Complexity: Every node decides $O(\log^2 n)$ time-slots after its wake-up whether or not it joins the MIS!

Overview

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MIS Algorithm - Overview

<u>Goal of first phase</u>: Determine a limited number of *candidates*

- Upon waking up do:

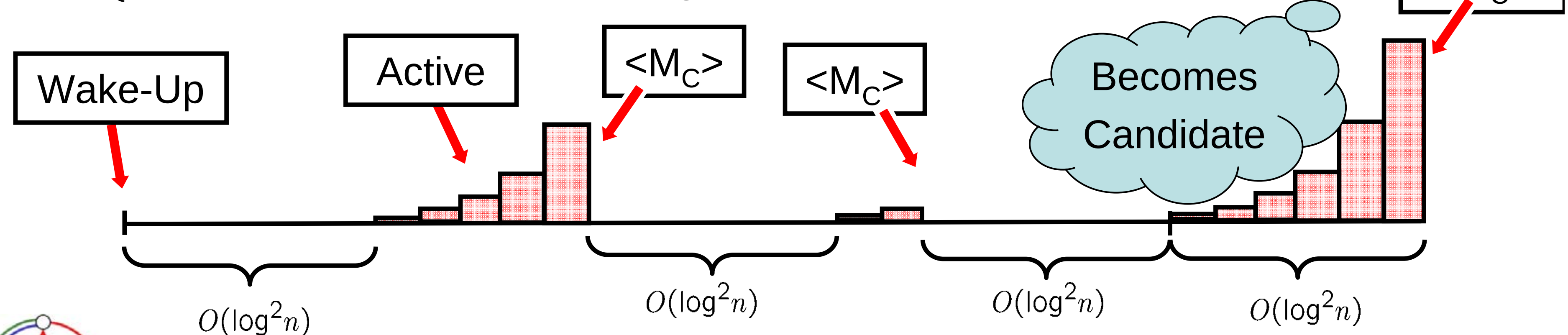
1) For $O(\log^2 n)$ time-slots do
   - If receive message from MIS node $\rightarrow$ stop
   - If receive message $<M_C>$ from active node $\rightarrow$ restart phase 1

Initial waiting phase

2) For i=0 to log n do
   - Set sending probability $p_i := C*2^i/n$
   - For $\lambda \log n$ time-slots do
     - Send message $<M_C>$ with probability p_i $\rightarrow$ become Candidate
     - If receive message from MIS node $\rightarrow$ stop
     - If receive message $<M_C>$ from active node $\rightarrow$ restart phase 1

Active Node

Exponentially increase sending probability

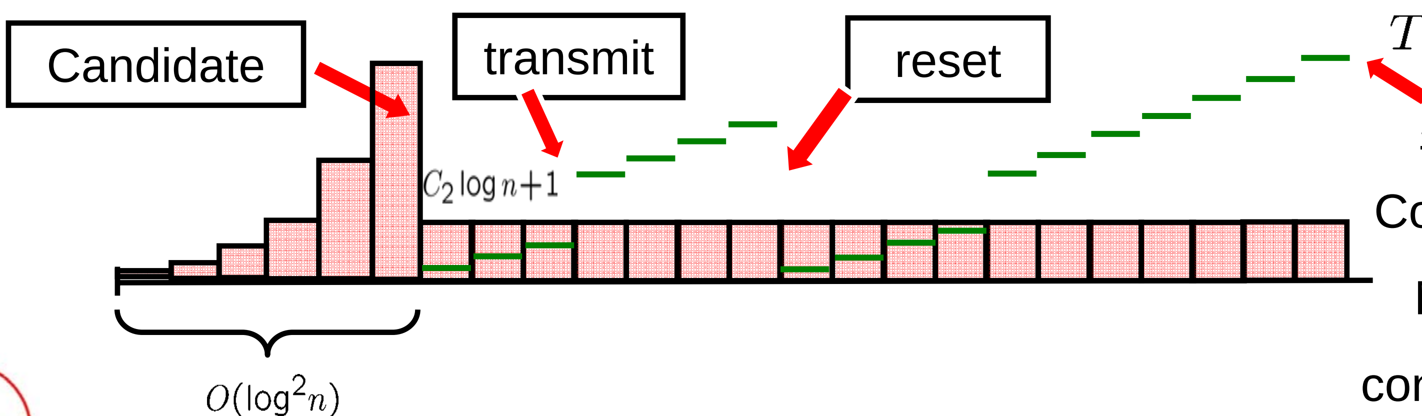


MIS Algorithm - Overview

<u>Goal of second phase</u>: Choose MIS Nodes from among candidates.

- Upon becoming candidate do:
- Each candidate has a local counter, initialized to 0.

3) In every time-slot
   - Transmit the local counter with probability $p \in \Theta(\log^{-1} n)$
     → set local counter to max{counter, $C_2 \cdot \log n+1$}
   - If receive message with neighboring counter
     AND if difference between counters is at most $C_2 \cdot \log n$
     THEN reset local counter to 0
   - If receive message from MIS node → stop
   - Increase counter by 1, if it reaches a threshold $T \in O(\log^2 n)$, join MIS



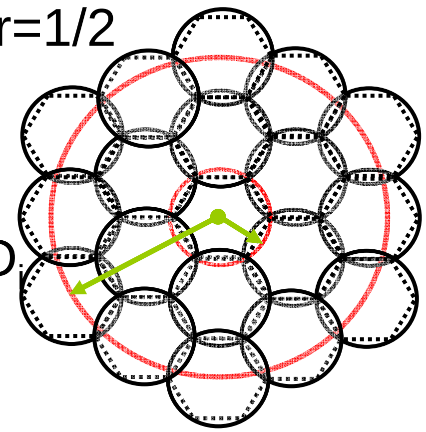
<u>Join the MIS:</u>

Continue sending Message with constant probability

MIS Algorithm – Basic Proof Idea

(1) Sum of sending probabilities
(2) Number of candidates
(3) Independence

- Cover the plane with (imaginary) disks D_i of radius r=1/2
- Let E_i be the disk with radius R=3/2
- A node in D_i can hear all nodes in D_i
- Nodes outside of E_i cannot interfere with nodes in D_i



Throughout the execution, three properties hold w.h.p. (probabilistic invariants)

(1) The *sum of sending probabilities* $s(t) := \sum_{k \in C_i} p_k(t)$ in a disk D_i is bounded by a constant α.

(2) The *number of candidates* in any disk D_i is at most O(log n).

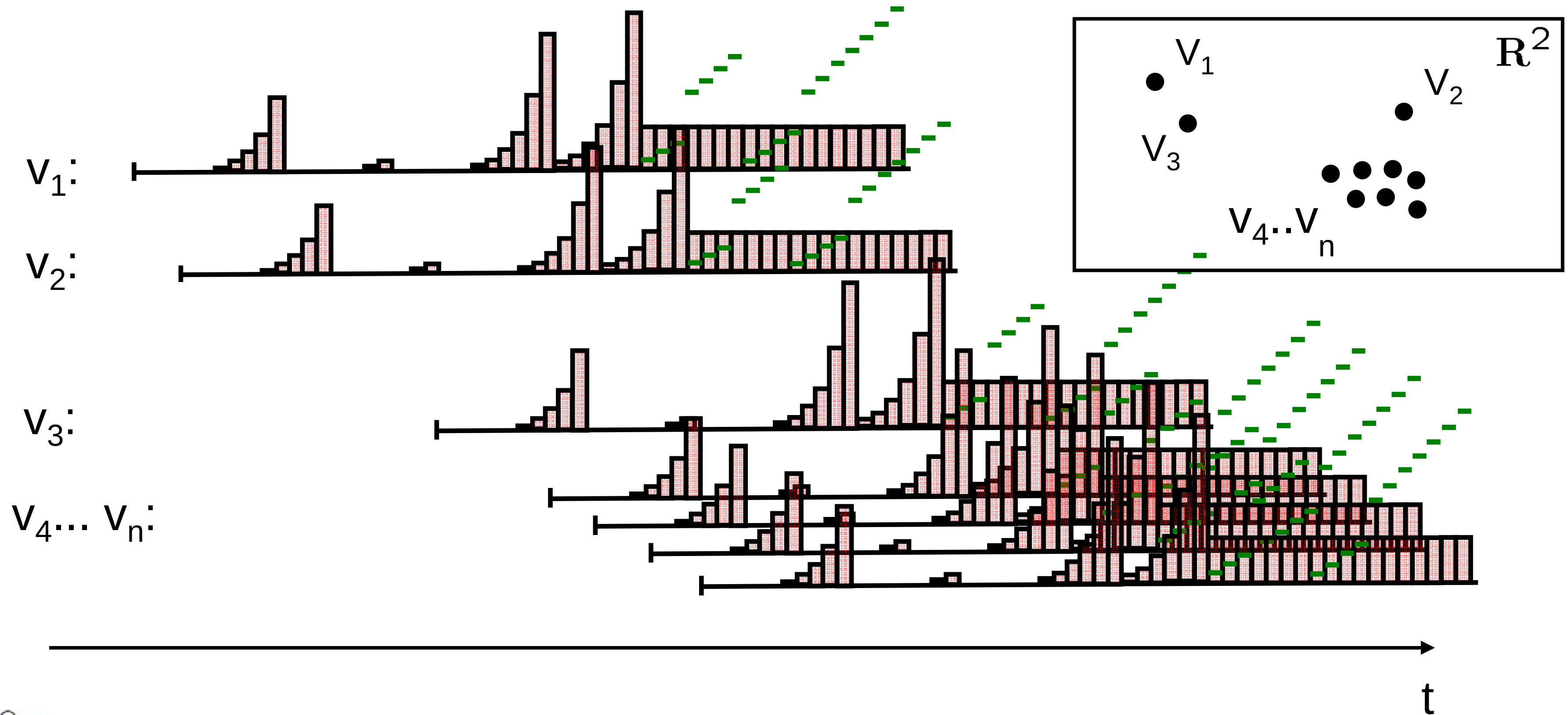
(3) The set of MIS nodes is *independent*

- At the beginning, all properties hold.
- Show that w.h.p., none of the properties is violated first.

Proof : (2)+(3) → (1)

(1) Sum of sending probabilities
(2) Number of candidates
(3) Independence

- Assume (2), (3) hold → what is the probability of (1) being violated?
- The proof is made difficult by asynchronous wake-up
- Executions of different nodes can be arbitrarily shifted in time

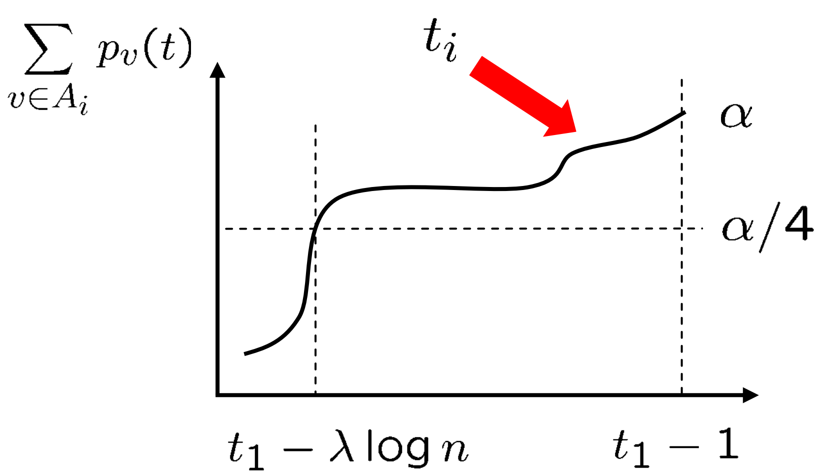


(1) Sum of sending probabilities
(2) Number of candidates
(3) Independence

Proof : (2)+(3) → (1)

- Assume (1) is *first* property to be violated at time t_1 !
- Consider λlog n time-slots betore t_1.
  - (2) and (3) hold in time-slots $t_1 - \lambda \log n, \ldots, t_1 - 1$
  - Every active node at time $t_1 - \lambda \log n$ will its probability exactly once
  - At most n new node may join with the initial sending probability

→ In interval $\mathcal{J} = t_1 - \lambda \log n, \ldots, t_1 - 1$, it holds that $\frac{1}{4}\alpha \leq \sum_{v \in A_i} p_v(t) \leq \alpha$



Consider an arbitrary time-slot $t_i \in \mathcal{J}$

Proof : (2)+(3) → (1)

(1) Sum of sending probabilities
(2) Number of candidates
(3) Independence

- If exactly one active node $v_a \in D_a$ transmits and no other node in E_a
  → *Clearance* in D_a occurs
  → v_a becomes candidate, all other active nodes restart

Sending probability reduces to 0 in D_a

→ Probability of a clearance of D_a in $t_i \in \mathcal{J}$:

$$
\begin{aligned}
P_c(t_i) &\geq P[\text{one active node sends in } D_a] \cdot P[\text{no other node sends in } E_a] \\
&\geq \sum_{v \in \mathcal{A}_a} \left(p_v \prod_{w \in D_a \setminus \{v\}} (1 - p_w) \right) \cdot \prod_{q \in E_a} (1 - p_q) \\
&\geq \sum_{v \in \mathcal{A}_a} p_v \cdot \left(\frac{1}{4}\right)^{\sum_{w \in D_a} p_w} \cdot \left(\left(\frac{1}{4}\right)^{\sum_{w \in D_a} p_w} \right)^{O(1)} \geq \Omega(1).
\end{aligned}
$$

bounded by constant

O(1) if (2) and (3) hold

Wait! There can be infinitely many t_i → high probability?

→ There is a clearance in $\mathcal{J}$ w.h.p.

We show that unless (1) is violated, there are at most O(1) clearances in D_a.

Hence, (1) is not violated unless (2) or (3) is violated first w.h.p.

Proof : (1)+(3) → (2)

(1) Sum of sending probabilities
(2) Number of candidates
(3) Independence

- Assume (1), (3) hold → what is the probability of (2) being violated?
- We show that the number of new candidates emerging in D_a is at most O(log n) between two clearances!

Definition: A time-slot is a *failure* in D_a if a new candidate emerges, but there is no clearance of D_a.

- We prove: The number of failures between two clearances is bounded by O(log n) w.h.p.
- Consider a time-slot in which at least one active node sends:

$$P_t[\text{clearance}|\text{at least one node sends}] \geq \ldots \geq \Omega(1)$$

Geometric Series:
At most O(log n) failures

(1) Sum of sending probabilities
(2) Number of candidates
(3) Independence

Proof : (1)+(3) → (2)

- How many candidates can emerge during these O(log n) failures?

- In expectation, O(1) sending nodes (=new candidates) per failure.
- If there are at most O(log n) failures, then O(log n) new candidates in expectation.

- Using Chernoff Bound → At most O(log n) new candidates between two subsequent clearances w.h.p
  (*if* number of failures bounded by O(log n))

Unless property (2) is violated, there are at most O(1) clearances in D_a.

Hence, (2) is not violated unless (1) or (3) is violated first w.h.p.

Proof : (1)+(2) → (3) (i.e. Correctness)

(1) Sum of sending probabilities
(2) Number of candidates
(3) Independence

- Assume (1), (2) hold → Will the MIS be correct?
- We prove:

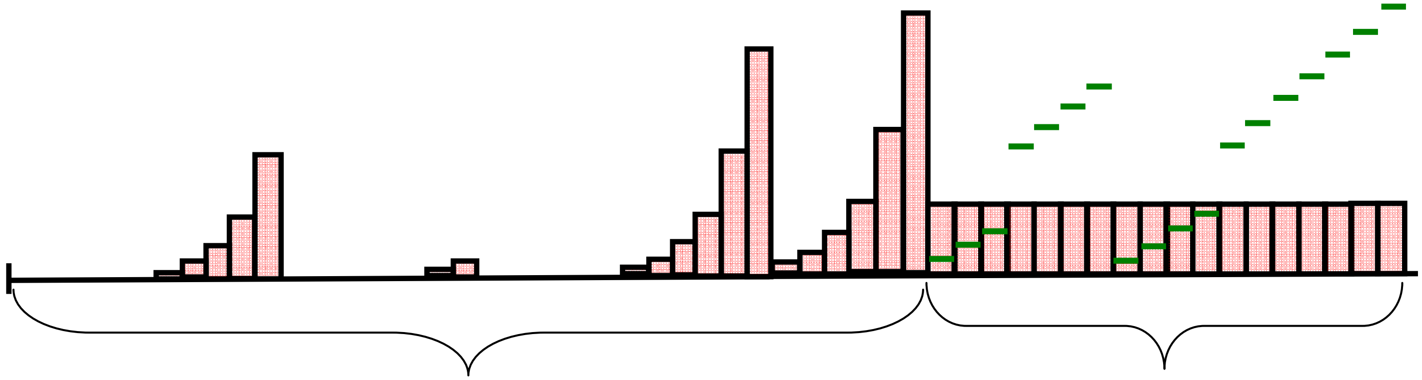
1) No two neighbors join MIS within O(log n) time w.h.p!
→ Proof by contradiction:
Consider $O(\log^2 n)$ time-slots before the first joined MIS.
We show:
second node receives message from first
→ second node resets its counter!

2) The first MIS node will successfully send a MIS message to all its neighbors within time O(log n) after joining MIS!

3) It follows: (3) holds → MIS is correct w.h.p. !

Proof : Running Time...

- How long does a node need to wait...?



How many times must a node restart?

How many times must a candidate reset?

→ <u>We prove:</u>

After at most $O(\log^2 n)$ slots, v is either covered or becomes a candidate!

→ <u>We prove:</u>

After at most $O(\log^2 n)$ slots, v is either covered or joins the MIS!

Every node decides within time $O(\log^2 n)$, w.h.p

Conclusions / Open Problems

We believe $O(\log^2 n)$ to be tight.

- MIS in harsh radio network model in time $O(\log^2 n)$ w.h.p.
  → Tight up to a factor of O(loglog n)
  → Compare to Luby's O(log n) in message passing models
  → Both algorithms are known to be close from optimal

- Future Directions / Open Problems:
  - Tighten upper or lower bound
  - Study the complexity of other local network structures in radio networks
  - Deterministic constructions?

Questions? Comments?

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