Public Advertisement Broker Markets.

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Abstract. Motivated by the growth of various networked systems as potential market places, we study market models wherein, owing to the size of the markets, transactions take place between largely unknown agents. In such scenarios, intermediaries or brokers play a significant role in a transaction.
We analyze market behavior in large networks wherein all sellers are not known to the buyers and vice-versa and depend on intermediaries to conduct any transactions. In such markets, we study a specific case where buyers wish to purchase goods from trusted sources at minimal prices. Sellers wish to maximize selling price. Brokers attempt to maximize profit by aiding in trade by acting as intermediaries; brokers have an advertising budget. We show the existence of competitive equilibria in such layered broker markets. We also describe efficient algorithms to compute these equilibria. We give polynomial-time distributed mechanisms to reach the equilibrium for two extreme cases of the brokers' advertising budget constraints.

1 Introduction

The large size and complexity of many markets necessitates the existence of intermediaries or brokers who mediate transactions between buyers and sellers. The large size of a market often results in the development of varying degree of personal relationships between all the players in the market. For example, a buyer can prefer to deal with one broker over another when she chooses to buy a certain commodity from the market. For example, in bilateral search markets (e.g., employment agencies, real estate brokers), the middleman narrows the set of buyers and sellers who search. In such markets, sellers with high selling prices and buyers with small budgets drop out of the search market and instead trade through the middleman [12].

With the growth of technology, in particular the Internet, markets have changed drastically from their traditional bearings,especially in scale, and the need for understanding the role of the middleman in these new settings gains ever more significance. With easy access to information about commodities, scenarios arise wherein multiple brokers often compete to sell the same commodity to the buyer. In such cases, they differentiate themselves not only using pricing discounts but also other value added services to cultivate a longer term relationship with the buyer. Therefore, it is not only important *what* you get, but also *who* you get from. All the factors above lead to a slew of interesting questions like how to model a market, and best behavior strategies in certain models.

In this study, we model various agents being nodes of a directed network. This network is defined for a specific good in the market as the trust network and market behavior depends on the good. Inter-play between dependent goods is a tangential albeit very interesting and important study, that we do not explore in this paper. For any good, each node plays the role of either a buyer, a seller or a broker. Loosely, they correspond to people who need the good and are willing to pay, who have the good and wish to sell, and the intermediaries who make money due to their important role of being a middleman. The model is motivated by the following observations.

- A buyer wishes to maximize his happiness. The happiness not only depends on the good she gets and the price she pays for it, but also depends crucially on who she gets its from.
- A broker wishes to maximize her *profit*. Thus, it is important for her, given her constraints, to decide what to buy, from whom to buy, and for how much, and whom to sell it to.
- A seller wishes to sell at the maximum possible price. These prices depend on the quality of the good and the reputation of the seller.

Our Results

In this paper, we look at networks where buyers and sellers form the extreme layers, with brokers in between. We show that competitive equilibria exist in these networks, and give polynomial time algorithms for computing them. In certain restricted settings, we give efficient mechanisms to reach the equilibrium.

We should point out that our notion of equilibrium differs from the market equilibrium concept of Arrow and Debreu [1]. We consider *indivisible* goods, and not much is known in the Arrow-Debreu setting for indivisible goods, even in the case with no brokers. Our equilibrium notion is that of a competitive equilibrium: buyers get their best goods at the prices, sellers sell at optimum prices, and brokers have no envy or regret in their dealings.

In the case of no brokers, the model above reduces to the market generated by the assignment game of Shapley and Shubik [11], who show the existence of competitive equilibria in these markets. Efficient algorithms for the same are implied by various later works [3], etc.

Related Work Motivated by the indirect interaction among agents, in their paper, Graphical Economies, Kakade et.al [5] introduce a graph-theoretic generalization of the classical Arrow-Debreu economics. They provide existence results for market equilibria and give algorithms for the same. Rubenstein and Wolinsky [9] study a market model which includes intermediaries and analyzes steady state conditions in such markets. An excellent survey is due to Jackson [4]. Recently, Kleinberg and Raghavan [6] considered query-incentive networks to theoretically study the effect of incentive issues in networks.

Our work was done independent of a recent paper by Blume et. al. [7]. They study a very similar problem of the interaction of buyers and sellers through a layer of intermediaries. In their model, the intermediaries set prices for both sellers and buyers. On the contrary, in our case, sellers *advertise* prices that brokers consume and in turn advertise prices for buyers. Further, the techniques used in their results are LP based which is an alternate way of looking at the initial assignment game of [11]. The techniques in our paper are more algorithmic. We prove the existence of *envy-free* Nash equilibria in our setting, and for certain restricted cases, provide efficient distributed mechanisms to reach the same.

Our model is more general than the ones considered by Babaioff, Walsh [2] and Babaioff, Nisan, Pavlov [8]. The latter paper assumes that the products of the sellers are indistinguishable for the buyers; in contrast our paper allows buyers to have preferences over the sellers and the brokers.

2 Broker Market Games

We model the three kinds of players in our market: buyers, brokers and sellers, as the three layers in a tri-partite network (A, B, C), respectively. Thus we allow interaction between buyers and sellers only via brokers. We assume each seller has one good to sell and each buyer desires only one unit of good. We also assume the same number of buyers and sellers, i.e. $|A| = |C| = n$ and $|B| = m$. Every buyer i has a value u_i associated on obtaining the good. Every seller j has a global reputation r_j. The buyer i to broker j trust weights are denoted by α_{ij}.

We consider the constrained advertisement model for the brokers, that is, we allow each broker j a maximum of N_j advertisements which he can broadcast. For simplicity we assume each $N_j = N$ is the same, and we assume $Nm \geq n$.

We now describe the strategies of the various players and the payoffs they get.

- *Buyers:* The strategy of the buyer is to decide which broker j to trade with, to buy the item from which seller k, when the price the broker offers is p_j. The pay-off is a function $f_i(u_i, \alpha_{ij}, p_j, r_k)$ which is assumed to be continuous, increasing in u_i, α_{ij} and r_k, and decreasing in p_j. We will assume its is linear in the price. The pay-off f_i is called *separable* if there exist functions g_i and h such that
$$f_i(u_i, \alpha_{ij}, p_j, r_k) = g_i(u_i, \alpha_{ij}) + h(r_k) - p_j$$
- *Brokers:* The broker j need to decide which of the N sellers' advertisements should it broadcast and at what prices. The only profit broker makes is $P_{sell} - P_{buy}$, where P_{sell} is the total price he sells goods at and P_{buy} the total money he buys at. For this paper we assume brokers do not make distinctions between which buyers he sells to.
- *Sellers:* The strategy of the seller is to just fix the price at which he sells the good.

Under the assumptions stated above, this game has a Nash equilibrium in pure strategies. We first elaborate what a PSNE looks like in this case.

PSNE (Pure Strategy Nash Equilibrium) of Broker Market Games: A price vector P for sellers, an allocation of some N advertisements for every broker j, and a price vector Q for the mN advertisements by brokers form a Nash equilibrium if the following hold.

- Buyers get the best possible product at the given prices, i.e., at the price vector Q, every buyer i buys a good from seller k advertised by broker j at price Q_j such that it maximizes $f_i(u_i, \alpha_{ij}, Q_j, r_k)$.
- Every good is sold.
- Sellers have no incentive to raise advertised prices, i.e., increase in $p_k = P[k]$ results in seller k's good having zero demand.
- All m brokers have *zero-regret* about the advertisements they chose to broadcast, that is, no broker could have made more profit by broadcasting a different set of N advertisements at some other price.

We further define the following desirable envy-freeness property and then a result on existence of PSNE in broker markets. We defer the proof to the full version [10].

Definition 1. *Envy-freeness of brokers: Broker s will not envy broker t only if every buyer i broker t sold a good to with > 0 profit, is either* happier *buying that good from t than from s, or s did not advertise that good.*

Theorem 1. *For every broker market game with separable pay-off functions for buyers, there exists an envy-free PSNE.*

Our model can be thought off as a generalization of the *assignment game*, defined by Shapley and Shubik [11] which comprised of n buyers and n sellers with each buyer i having a utility u_{ij} for the good sold by j. Their goal was to come up with a price vector P for the goods, such that at this price, every buyer i gets the good maximizing her "happiness" of $u_{ij} - p_j$ and the market clears. Demange et.al. [3] came up with a mechanism to reach these equilibrium prices.

In the next section, we discuss cases where the PSNE can be reached in an efficient fashion via distributed mechanisms.

2.1 Efficient Mechanisms

For certain special cases, we also design efficient mechanisms for computing these envy-free Nash equilibria. Due to lack of space, we only describe one such mechanism.

Theorem 2. *For the extreme cases when $N = n$ (unbounded advertising budget) and $N = 1$ (one advertisement budget), there exist polynomial time distributed mechanisms to compute the Nash equilibria.*

Unbounded Advertising Budget The mechanism can be thought of as a sequence of two mini-games played one after another until all goods are sold. Both mini-games are similar to the assignment game. We now sketch a continuous version of the mechanism and we leave out the discretization and proof of polynomial time in this abstract.

THE MULTIPLE ROUND MECHANISM
Initialize $P = 0$ for all sellers.
MINI GAME 1

- Sellers pass price-vector P to all brokers.
- Brokers pass P to all buyers they can reach.
- Buyers choose their best advertisement(s) from their options. Draw corresponding $\geq n$ edges.
- As long as there is a subset of buyers, S, such that $N(S)$: the set of brokers they choose as their best option, is smaller in size than S, brokers in $N(S)$ corresponding to a maximal such S increase their selling prices.
- If for all $S \subseteq A$, $|N(S)| \geq |S|$, then by Hall's theorem there is a matching which matches all buyers. Choose such an arbitrary matching and call the matched brokers *active*, and move on to Mini-game 2.

MINI GAME 2

- Draw edges from *active* brokers to sellers that received demand from buyers.
- Consider a maximal subset T of (active) brokers such that the size of its neighborhood $|N(T)| < |T|$, if such a T exists. Repeat this and the following step until no such T exists.
- Sellers in $N(T)$ corresponding to a maximal set T increase their price, modifying P. As soon as some seller's selling price reaches a neighboring broker's selling price, delete the broker-seller edge.
- Brokers advertise this new vector P and a second round of mini-games takes place unless $|N(T)| = |C|$, that is all goods are sold.

3 Concluding Remarks

There are several interesting future directions. We prove the envy-freeness property for the specific Nash equilibria we describe. A fundamental question is to give a clean characterization of all possible Nash equilibria in our setting. It is not clear whether one can extend the efficient distributed mechanisms to the intermediate budget constraint cases.

The eventual goal of our study is to prove existence, determine efficient mechanisms, and characterize equilibria in general networks (not necessarily layered). One question about the model that merits attention is that in our case we allow brokers to choose which sellers to advertise. An alternate model could allow sellers to choose which brokers to advertise via; in such a setting, it may make sense to attribute global (perhaps dynamic) reputations on brokers too.

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